CollegeBoard

AP® Calculus AB Practice Exam

From the 2015 Administration

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Note: This publication shows the page numbers that appeared in the *2014–15 AP Exam Instructions* book and in the actual exam. This publication was not repaginated to begin with page 1.

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Exam Instructions

The following contains instructions taken from the *2014–15 AP Exam Instructions* book.

AP® Calculus AB/BC Exam

Regularly Scheduled Exam Date: Tuesday morning, May 5, 2015

Late-Testing Exam Date: Thursday morning, May 21, 2015

Section I Total Time, Calculus AB: 1 hr. 45 min. Section II Total Time, Calculus AB: 1 hr. 30 min.
Section I Total Time, Calculus BC: 1 hr. 45 min. Section II Total Time, Calculus BC: 1 hr. 30 min.

Section I	**Total Time:** 1 hour 45 minutes **Number of Questions:** 45* **Percent of Total Score:** 50% **Writing Instrument:** Pencil required **The number of questions may vary slightly depending on the form of the exam.*	**Part A:** **Number of Questions:** 28 **Time:** 55 minutes *No calculator allowed*	**Part B:** **Number of Questions:** 17 **Time:** 50 minutes *Graphing calculator required*
Section II	**Total Time:** 1 hour 30 minutes **Number of Questions:** 6 **Percent of Total Score:** 50% **Writing Instrument:** Either pencil or pen with black or dark blue ink **Note**: For Section II, if students finish Part A before the end of the timed 30 minutes for Part A, they cannot begin working on Part B. Students must wait until the beginning of the timed 60 minutes for Part B. However, during the timed portion for Part B, students may work on the problems in Part A without the use of a calculator.	**Part A:** **Number of Questions:** 2 **Time:** 30 minutes **Percent of Section II Score:** 33.3% *Graphing calculator required*	**Part B:** **Number of Questions:** 4 **Time:** 60 minutes **Percent of Section II Score:** 66.6% *No calculator allowed*

What Proctors Need to Bring to This Exam

- Exam packets
- Answer sheets
- AP Student Packs
- *2014-15 AP Coordinator's Manual*
- This book — *AP Exam Instructions*
- AP Exam Seating Chart template(s)
- School Code and Home-School/Self-Study Codes
- Extra graphing calculators
- Pencil sharpener
- Container for students' electronic devices (if needed)
- Extra No. 2 pencils with erasers
- Extra pens with black or dark blue ink
- Extra paper
- Stapler
- Watch
- Signs for the door to the testing room
  - "Exam in Progress"
  - "Cell phones are prohibited in the testing room"

SEATING POLICY FOR AP CALCULUS AB AND CALCULUS BC EXAMS

Testing Window	Exams Administered at Schools in the United States, Canada, Puerto Rico, and the U.S. Virgin Islands	Exams Administered at Schools Outside the United States, Canada, Puerto Rico, and the U.S. Virgin Islands
Regularly Scheduled Exams	**Students must be seated no less than four feet apart.**	**Students must be seated no less than five feet apart.**
Late-Testing Exams	**Students must be seated no less than five feet apart.**	

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Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 46 of the *2014-15 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 44 of the *2014-15 AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.**

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time. Remember: You must complete a seating chart for this exam. See pages 279–280 for a seating chart template and instructions. See the *2014-15 AP Coordinator's Manual* for exam seating requirements (pages 48–50, 88).

If you are giving the regularly scheduled exam, say:

It is Tuesday morning, May 5, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the alternate exam for late testing, say:

It is Thursday morning, May 21, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

In a moment, you will open the packet that contains your exam materials.

By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2014-15 Bulletin for AP Students and Parents*. Please check to make sure you have the correct exam: Calculus AB or Calculus BC. Raise your hand if you do not have the correct exam. . . .

You may now remove the shrinkwrap from your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. . . .

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Carefully remove the AP Exam label found near the top left of your exam booklet cover. Now place it on page 1 of your answer sheet on the light blue box near the top right-hand corner that reads "AP Exam Label."

If students accidentally place the exam label in the space for the number label or vice versa, advise them to leave the labels in place. They should not try to remove the label; their exam will be processed correctly.

Read the statements on the front cover of Section I and look up when you have finished. . . .

Sign your name and write today's date. Look up when you have finished. . . .

Now print your full legal name where indicated. Are there any questions? . . .

Turn to the back cover and read it completely. Look up when you have finished. . . .

Are there any questions? . . .

You will now take the multiple-choice portion of the exam. You should have in front of you the multiple-choice booklet and your answer sheet. You may never discuss these specific multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled.

You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses beginning on page 2 of your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.

Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .

You have 55 minutes for Part A. Part A questions are numbered 1 through 28. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.

Note Start Time here _____. Note Stop Time here _____. Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 45 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working on Part A and turn to page 22 in your Section I booklet. . . .

On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold

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the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 92. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 50 minutes for Part B. You may begin.

Note Start Time here _____. Note Stop Time here _____. Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other. After 40 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working and turn to page 38. You have 3 minutes to answer Questions 93–96. These are survey questions and will not affect your score. You may not go back to work on any of the exam questions. . . .

Give students approximately 3 minutes to answer the survey questions. Then say:

Close your booklet and put your answer sheet on your desk, face up. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. Sit quietly while I collect your answer sheets.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label. After all answer sheets have been collected, say:

Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, face up. I will now collect your Section I booklet. . . .

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, or textbooks during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you may never discuss the

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multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled. Are there any questions? . . .

You may begin your break. Testing will resume at _____.

SECTION II: Free Response

After the break, say:

May I have everyone's attention? Place your Student Pack on your desk. . . .

You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .

Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .

Now place an AP number label on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using your pen, print the first, middle and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and complete Item 1 under "Important Identification Information." Print the first two letters of your last name and the first letter of your first name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

I need to collect the Student Pack from anyone who will be taking another AP Exam. You may keep it only if you are not taking any other AP Exams this year. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .

While Student Packs are being collected, read the information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs. Then say:

Are there any questions? . . .

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Section II also has two parts that are timed separately. You are responsible for pacing yourself, and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time. Are there any questions? . . .

You have 30 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra sheet of paper you use, be sure to write only your AP number and the number of the question you are working on. Do not write your name. Open your exam booklet and begin.

Note Start Time here _____. Note Stop Time here _____. Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1s or 2s at the top of each page to assist you in monitoring their work. After 20 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .

Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may not use your calculator. Remember to show your work, and write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .

Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may begin Part B.

Note Start Time here _____. Note Stop Time here _____. After 50 minutes, say:

There are 10 minutes remaining in Part B.

After 10 minutes, say:

Stop working and close your exam booklet. Place it on your desk, face up. . . .

If any students used extra paper for the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their exam booklets. Complete an Incident Report and include any exam booklets with extra sheets of paper in an Incident Report return envelope (see page 57 of the *AP Coordinator's Manual* for details). Then say:

Remain in your seat, without talking, while the exam materials are collected. . . .

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box, and printed his or her initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

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When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:

You may not discuss or share these specific free-response questions with anyone unless they are released on the College Board website in about two days. Your AP Exam score results will be available online in July.

If you are giving the alternate exam for late testing, say:

None of the questions in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

All exam materials must be placed in secure storage until they are returned to the AP Program after your school's last administration. Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:

- Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See "Post-Exam Activities" in the *2014-15 AP Coordinator's Manual*.
- Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Be sure to give the completed seating chart to the AP Coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

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Student Answer Sheet for the Multiple-Choice Section

Use this section to capture student responses. (Note that the following answer sheet is a sample, and may differ from one used in an actual exam.)

AP Answer Sheet 2015

AP Number Label
(from Student Pack)

B123456789T

AP Exam Label
(from Section I Booklet)

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COMPLETE THIS AREA AT EVERY EXAM. **USE NO. 2 PENCIL ONLY**

To maintain the security of the exam and the validity of my AP score, I will allow no one else to see the multiple-choice questions. I will seal the multiple-choice booklet when asked to do so, and I will not discuss these questions with anyone at any time after completing the section. I am aware of and agree to the AP Program's policies and procedures as outlined in the *2014-15 Bulletin for AP Students and Parents*, including using testing accommodations (e.g., extended time, computer, etc.) only if I have been preapproved by College Board Services for Students with Disabilities.

A. SIGNATURE Sign your legal name as it will appear on your college applications. Date

B. LEGAL NAME Omit apostrophes, Jr., II.

Legal Last Name — First 15 Letters | Legal First Name — First 12 Letters | MI

C. YOUR AP NUMBER

D. EXAM DATE Month | Day

E. EXAM START TIME AM | PM

F. MULTIPLE-CHOICE BOOKLET SERIAL NUMBER S

G. ONLINE PROVIDER CODE

COMPLETE THIS AREA ONLY ONCE.

I. AREA CODE AND PHONE NUMBER

INTERNATIONAL PHONE

J. SCHOOL YOU ATTEND

SCHOOL CODE | School Name | City | State | Country

K. DATE OF BIRTH

Month: Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec | Day | Year

L. SOCIAL SECURITY NUMBER (Optional)

M. COLLEGE TO RECEIVE YOUR AP SCORE REPORT

COLLEGE CODE

Using the college code listed in the AP Student Pack, indicate the **ONE** college that you want to receive your AP score report.

College Name | City | State | Country

N. CURRENT GRADE LEVEL

- Not yet in 9th grade
- 9th
- 10th
- 11th
- 12th
- No longer in high school

H. AP EXAM I AM TAKING USING THIS ANSWER SHEET Exam Name: | Form: | Form Code:

779934

SCHOOL USE ONLY

Section Number: 1 2 3 4 5 6 7 8 9

Fee Reduction Granted: 1 Option 1 2 Option 2

104246-00657 • UNLWEB315

Q3914/1-4

COMPLETE THIS AREA AT EACH EXAM (IF APPLICABLE).

O. SURVEY QUESTIONS — Answer the survey questions in the AP Student Pack. Do not put responses to exam questions in this section.

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ	4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ	7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ	5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ	8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ	6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ	9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ

P. LANGUAGE — Do not complete this section unless instructed to do so.

If this answer sheet is for the French Language and Culture, German Language and Culture, Italian Language and Culture, Spanish Language and Culture, or Spanish Literature and Culture Exam, please answer the following questions. Your responses will not affect your score.

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken?

◯ Yes ◯ No

2. Do you regularly speak or hear the language at home?

◯ Yes ◯ No

QUESTIONS 1–75

Indicate your answers to the exam questions in this section (pages 2 and 3). Mark only one response per question for Questions 1 through 120. If a question has only four answer options, do not mark option E. Answers written in the multiple-choice booklet will not be scored.

COMPLETE MARK ● EXAMPLES OF INCOMPLETE MARKS

You must use a No. 2 pencil and marks must be complete. Do not use a mechanical pencil. It is very important that you fill in the entire circle darkly and completely. If you change your response, erase as completely as possible. Incomplete marks or erasures may affect your score.

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	26 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	51 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	27 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	52 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	28 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	53 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	29 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	54 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	30 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	55 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	31 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	56 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	32 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	57 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	33 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	58 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	34 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	59 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	35 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	60 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	36 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	61 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
12 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	37 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	62 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	38 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	63 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
14 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	39 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	64 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	40 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	65 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
16 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	41 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	66 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	42 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	67 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
18 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	43 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	68 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	44 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	69 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
20 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	45 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	70 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	46 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	71 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
22 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	47 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	72 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	48 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	73 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
24 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	49 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	74 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	50 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	75 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

ETS USE ONLY

Exam		0 1 2 3 4 5 6 7 8 9
		0 1 2 3 4 5 6 7 8 9
Exam		0 1 2 3 4 5 6 7 8 9
		0 1 2 3 4 5 6 7 8 9

SELECTED MEDIA EXAMS	R	W	O	OTHER EXAMS	R	W	O
PT02				TOTAL			
PT03				Subscore (if applicable)			
PT04				Subscore (if applicable)			

DO NOT WRITE IN THIS AREA

QUESTIONS 76–120

Be sure each mark is dark and completely fills the circle. If a question has only four answer options, do not mark option E.

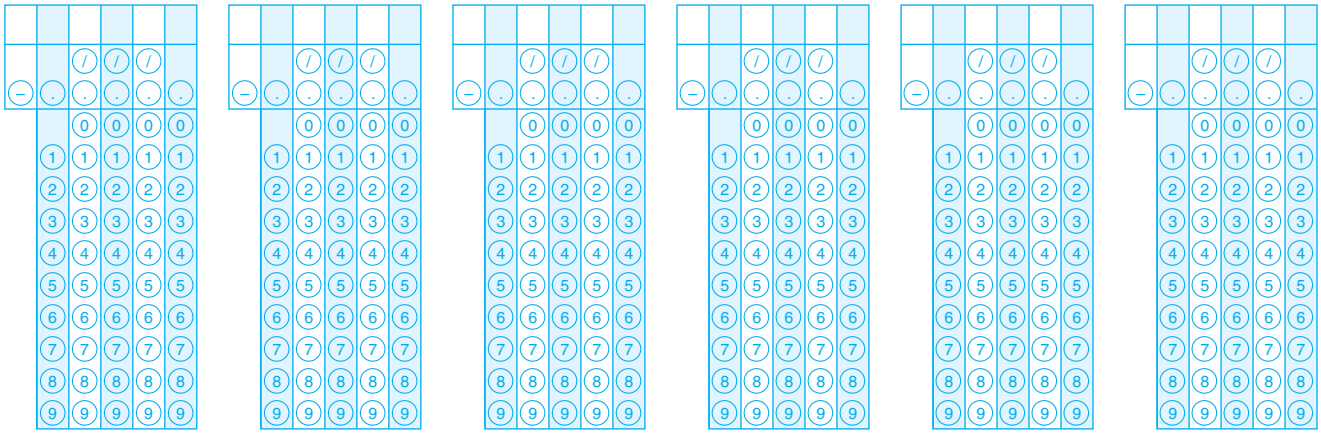
76	A B C D E	91	A B C D E	106	A B C D E
77	A B C D E	92	A B C D E	107	A B C D E
78	A B C D E	93	A B C D E	108	A B C D E
79	A B C D E	94	A B C D E	109	A B C D E
80	A B C D E	95	A B C D E	110	A B C D E
81	A B C D E	96	A B C D E	111	A B C D E
82	A B C D E	97	A B C D E	112	A B C D E
83	A B C D E	98	A B C D E	113	A B C D E
84	A B C D E	99	A B C D E	114	A B C D E
85	A B C D E	100	A B C D E	115	A B C D E
86	A B C D E	101	A B C D E	116	A B C D E
87	A B C D E	102	A B C D E	117	A B C D E
88	A B C D E	103	A B C D E	118	A B C D E
89	A B C D E	104	A B C D E	119	A B C D E
90	A B C D E	105	A B C D E	120	A B C D E

QUESTIONS 121–126

For Students Taking AP Biology

Write your answer in the boxes at the top of the griddable area and fill in the corresponding circles. Mark only one circle in any column. You will receive credit only if the circles are filled in correctly.

121 122 123 124 125 126



QUESTIONS 131–142

For Students Taking AP Physics 1 or AP Physics 2

Mark two responses per question. You will receive credit only if both correct responses are selected.

131	A B C D	135	A B C D	139	A B C D
132	A B C D	136	A B C D	140	A B C D
133	A B C D	137	A B C D	141	A B C D
134	A B C D	138	A B C D	142	A B C D

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DO NOT WRITE IN THIS AREA

COMPLETE THIS AREA ONLY ONCE.

Q. YOUR MAILING ADDRESS Use the address abbreviations from your AP Student Pack. Fill in only one circle per column. Indicate a space in your address by leaving a blank box; do not grid that column.

STREET ADDRESS (include street number, street name, apartment number, etc.)

CITY

ZIP OR POSTAL CODE

COUNTRY CODE

STATE

○ AK	○ HI	○ MI	○ NY	○ VT	
○ AL	○ IA	○ MN	○ OH	○ WA	
○ AR	○ ID	○ MO	○ OK	○ WI	
○ AZ	○ IL	○ MS	○ OR	○ WV	
○ CA	○ IN	○ MT	○ PA	○ WY	
○ CO	○ KS	○ NC	○ RI	○ Puerto Rico	
○ CT	○ KY	○ ND	○ SC		
○ DC	○ LA	○ NE	○ SD	○ AA	
○ DE	○ MA	○ NH	○ TN	○ AE	
○ FL	○ MD	○ NJ	○ TX	○ AP	
○ GA	○ ME	○ NM	○ UT	○ Other	
		○ NV	○ VA		

R. FOR STUDENTS OUTSIDE THE UNITED STATES ONLY

○ If your address does not fit in the spaces provided in Item Q, fill in as many circles as you can, then fill in the circle in Item R and print the remainder of your address in the spaces provided.

Address	City	State or Province	Country	ZIP or Postal Code

T. EMAIL ADDRESS By providing your email address, you are granting the College Board permission to use your email in accordance with the policies in the *2014-15 Bulletin for AP Students and Parents*.

S. STUDENT IDENTIFIER (Student ID Number)

U. STUDENT SEARCH SERVICE®

Colleges and scholarship programs may request your information to inform you of educational opportunities and financial aid.

Would you like us to supply your information?

○ Yes ○ No

If you don't answer and previously chose to participate in this service, we will continue providing your information.

V. SEX

○ Female ○ Male

W. WHICH LANGUAGE DO YOU KNOW BEST?

○ English
○ English and another language about the same
○ Another language

X. ETHNICITY/RACE

○ American Indian or Alaska Native
○ Asian, Asian American or Pacific Islander
○ Black or African American
○ Mexican or Mexican American
○ Puerto Rican
○ Other Hispanic, Latino or Latin American
○ White
○ Other

Y. PARENTAL EDUCATION LEVEL

In the first column, indicate the highest level of education of your parent/guardian. If you have two parents/guardians, indicate the level of education for your other parent/guardian in the second column. In the appropriate column for each parent/guardian, indicate whether this is your mother or female guardian or your father or male guardian.

○ Grade school	○ Grade school
○ Some high school	○ Some high school
○ High school diploma or equivalent	○ High school diploma or equivalent
○ Vocational or trade school	○ Vocational or trade school
○ Some college	○ Some college
○ Associate or two-year degree	○ Associate or two-year degree
○ Bachelor's or four-year degree	○ Bachelor's or four-year degree
○ Some graduate or professional school	○ Some graduate or professional school
○ Graduate or professional degree	○ Graduate or professional degree
○ Mother or female guardian	○ Mother or female guardian
○ Father or male guardian	○ Father or male guardian

Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2015 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

PLACE SEAL HERE

AP® Calculus AB Exam

SECTION I: Multiple Choice

2015

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

PLACE SEAL HERE

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4KBP6-S

66

PLACE SEAL HERE
DO NOT seal answer sheet inside

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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GO ON TO THE NEXT PAGE.

1. $\int \left(5e^{2x} + \frac{1}{x}\right) dx =$

(A) $\frac{5}{2}e^{2x} + \frac{2}{x^2} + C$

(B) $\frac{5}{2}e^{2x} + \ln|x| + C$

(C) $5e^{2x} + \frac{2}{x^2} + C$

(D) $5e^{2x} + \ln|x| + C$

(E) $10e^{2x} - \frac{1}{x^2} + C$

---

2. If $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$, then $f'(4) =$

(A) $\frac{1}{16}$ (B) $\frac{5}{16}$ (C) 1 (D) $\frac{7}{2}$ (E) $\frac{49}{4}$

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GO ON TO THE NEXT PAGE.

3. $\int x^2\left(x^3+5\right)^6\,dx =$

(A) $\frac{1}{3}\left(x^3+5\right)^6+C$

(B) $\frac{1}{3}x^3\left(\frac{1}{4}x^4+5x\right)^6+C$

(C) $\frac{1}{7}\left(x^3+5\right)^7+C$

(D) $\frac{3}{7}x^2\left(x^3+5\right)^7+C$

(E) $\frac{1}{21}\left(x^3+5\right)^7+C$

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GO ON TO THE NEXT PAGE.

x	0	25	30	50
$f(x)$	4	6	8	12

4. The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^{50} f(x)\,dx$ using the subintervals $[0, 25]$, $[25, 30]$, and $[30, 50]$?

(A) 290 (B) 360 (C) 380 (D) 390 (E) 430

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

5. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

(A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.

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6. Which of the following is an antiderivative of $3\sec^2 x + 2$?

(A) $3\tan x$ (B) $3\tan x + 2x$ (C) $3\sec x + 2x$ (D) $\sec^3 x + 2x$ (E) $6\sec^2 x \tan x$

---

7. If $f(x) = x^2 - 4$ and g is a differentiable function of x, what is the derivative of $f(g(x))$?

(A) $2g(x)$ (B) $2g'(x)$ (C) $2xg'(x)$ (D) $2g(x)g'(x)$ (E) $2g(x) - 4$

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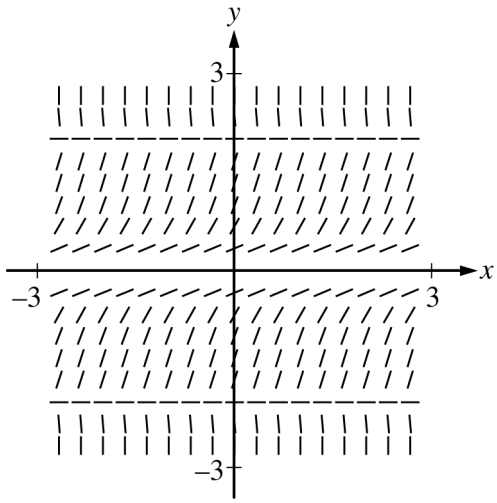

8. Shown above is a slope field for the differential equation $\frac{dy}{dx} = y^2\left(4 - y^2\right)$. If $y = g(x)$ is the solution to the differential equation with the initial condition $g(-2) = -1$, then $\lim_{x \to \infty} g(x)$ is

(A) $-\infty$ (B) -2 (C) 0 (D) 2 (E) 3

---

9. If $f''(x) = x(x+2)^2$, then the graph of f is concave up for

(A) $x < 0$

(B) $x > 0$

(C) $-2 < x < 0$

(D) $x < -2$ and $x > 0$

(E) all real numbers

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GO ON TO THE NEXT PAGE.

10. If $y = \sin x \cos x$, then at $x = \frac{\pi}{3}$, $\frac{dy}{dx} =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

---

11. $\lim_{x \to -3} \frac{x^2 - 9}{x^2 - 2x - 15}$ is

(A) 0 (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) 1 (E) nonexistent

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GO ON TO THE NEXT PAGE.

12. What is the average rate of change of $y = \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$?

(A) $-\frac{4}{\pi}$ (B) -1 (C) 0 (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{4}{\pi}$

---

13. If $y^3 + y = x^2$, then $\frac{dy}{dx} =$

(A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1 + 3y^2}$

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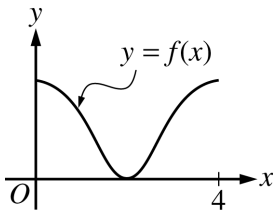
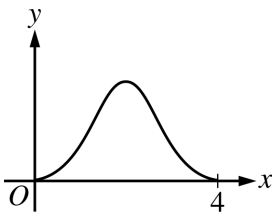
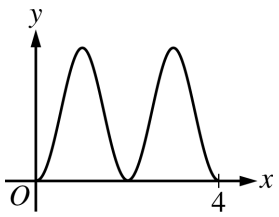
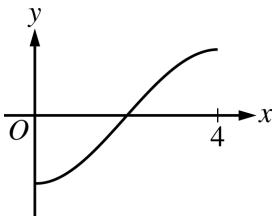
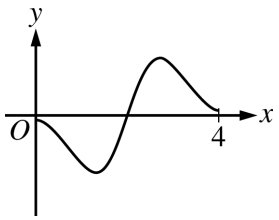


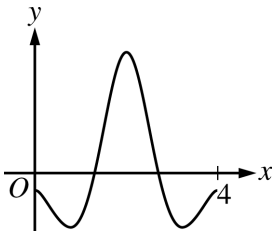
14. The graph of $y = f(x)$ on the closed interval $[0, 4]$ is shown above. Which of the following could be the graph of $y = f'(x)$?

(A)


(B)


(C)


(D)


(E)


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$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ \ln(3x - 2) & \text{if } x \geq 1 \end{cases}$$

15. Let f be the function defined above. Which of the following statements about f are true?

I. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

II. $\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

III. f is differentiable at $x = 1$.

(A) None

(B) I only

(C) II only

(D) II and III only

(E) I, II, and III

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16. The function f is defined by $f(x) = 2x^3 - 4x^2 + 1$. The application of the Mean Value Theorem to f on the interval $1 \le x \le 3$ guarantees the existence of a value c, where $1 < c < 3$, such that $f'(c) =$

(A) 0 (B) 9 (C) 10 (D) 14 (E) 16

---

17. The velocity v, in meters per second, of a certain type of wave is given by $v(h) = 3\sqrt{h}$, where h is the depth, in meters, of the water through which the wave moves. What is the rate of change, in meters per second per meter, of the velocity of the wave with respect to the depth of the water, when the depth is 2 meters?

(A) $-\frac{3}{4\sqrt{2}}$ (B) $-\frac{3}{8\sqrt{2}}$ (C) $\frac{3}{2\sqrt{2}}$ (D) $\frac{3}{\sqrt{2}}$ (E) $4\sqrt{2}$

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GO ON TO THE NEXT PAGE.

18. If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $20e^{-3}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

---

19. Let f be the function with derivative defined by $f'(x) = x^3 - 4x$. At which of the following values of x does the graph of f have a point of inflection?

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{3}$ (E) 2

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20. Let f be the function given by $f(x) = \dfrac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f, then $b =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

21. The base of a solid is the region bounded by the x-axis and the graph of $y = \sqrt{1-x^2}$. For the solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) $\dfrac{2}{3}$ (B) $\dfrac{4}{3}$ (C) 2 (D) $\dfrac{2\pi}{3}$ (E) $\dfrac{4\pi}{3}$

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22. Let f be the function given by $f(x) = \dfrac{kx}{x^2 + 1}$, where k is a constant. For what values of k, if any, is f strictly decreasing on the interval $(-1, 1)$?

(A) $k < 0$

(B) $k = 0$

(C) $k > 0$

(D) $k > 1$ only

(E) There are no such values of k.

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23. Which of the following is an equation for the line tangent to the graph of $y = 3 - \int_{-1}^{x} e^{-t^3}\,dt$ at the point where $x = -1$?

(A) $y - 3 = -3e(x + 1)$

(B) $y - 3 = -e(x + 1)$

(C) $y - 3 = 0$

(D) $y - 3 = \frac{1}{e}(x + 1)$

(E) $y - 3 = 3e(x + 1)$

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24. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 5y^2$ with the initial condition $y(0) = 3$?

(A) $y = \sqrt{9e^{5x}}$

(B) $y = \sqrt{\frac{1}{9}e^{5x}}$

(C) $y = \sqrt{e^{5x} + 9}$

(D) $y = \frac{3}{1 - 15x}$

(E) $y = \frac{3}{1 + 15x}$

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25. $\lim_{h \to 0} \dfrac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent

---

26. An object moves along a straight line so that at any time $t \geq 0$ its velocity is given by $v(t) = 2\cos(3t)$. What is the distance traveled by the object from $t = 0$ to the first time that it stops?

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{2}{3}$ (D) $\frac{\pi}{3}$ (E) $\frac{4}{3}$

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x	$f(x)$	$f'(x)$
0	49	0
1	2	–8
2	–1	–80

27. The table above gives selected values for a differentiable and decreasing function f and its derivative. If f^{-1} is the inverse function of f, what is the value of $\left(f^{-1}\right)'(2)$?

(A) -80 (B) $-\frac{1}{8}$ (C) $-\frac{1}{80}$ (D) $\frac{1}{80}$ (E) $\frac{1}{8}$

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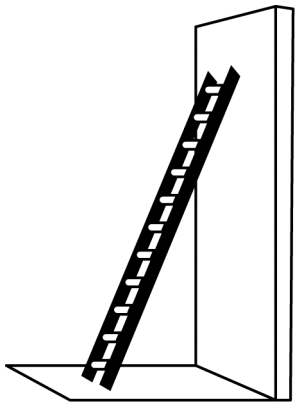

28. The top of a 15-foot-long ladder rests against a vertical wall with the bottom of the ladder on level ground, as shown above. The ladder is sliding down the wall at a constant rate of 2 feet per second. At what rate, in radians per second, is the acute angle between the bottom of the ladder and the ground changing at the instant the bottom of the ladder is 9 feet from the base of the wall?

(A) $-\frac{2}{9}$ (B) $-\frac{1}{6}$ (C) $-\frac{2}{25}$ (D) $\frac{2}{25}$ (E) $\frac{1}{9}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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PART B STARTS ON PAGE 24.

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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76. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1) = 0.076$?

(A) On February 1, 2010, the population of the world was increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world was increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world was 0.076 billion people.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world was 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

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x	0	2	4	6	8	10
$f(x)$	5	7	8	0	–15	–20

77. Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \le x \le 10$?

(A) -6 (B) $-\frac{5}{2}$ (C) -2 (D) $-\frac{2}{5}$ (E) $\frac{2}{5}$

---

78. The rate at which motor oil is leaking from an automobile is modeled by the function L defined by $L(t) = 1 + \sin\left(t^2\right)$ for time $t \ge 0$. $L(t)$ is measured in liters per hour, and t is measured in hours. How much oil leaks out of the automobile during the first half hour?

(A) 1.998 liters

(B) 1.247 liters

(C) 0.969 liters

(D) 0.541 liters

(E) 0.531 liters

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	4	2	π

79. The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$.

If $h(x) = \dfrac{f(x)}{g(x)}$, what is the value of $h'(0)$?

(A) $\dfrac{8 - 3\pi}{4}$ (B) $\dfrac{3\pi - 8}{4}$ (C) $\dfrac{4}{\pi}$ (D) $\dfrac{2 - 3\pi}{2}$ (E) $\dfrac{8 + 3\pi}{4}$

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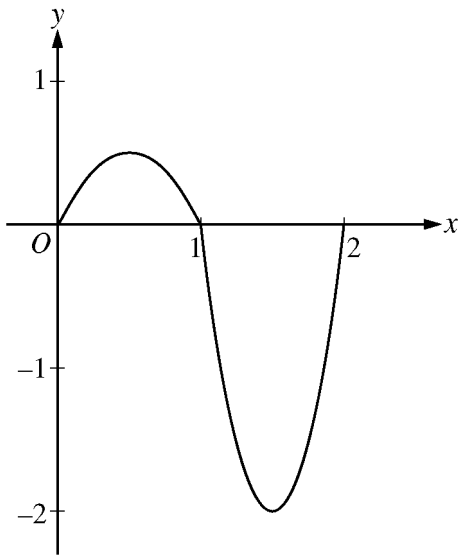

Graph of f'

80. The figure above shows the graph of f', the derivative of a function f, for $0 \le x \le 2$. What is the value of x at which the absolute minimum of f occurs?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) 2

---

81. What is the area of the region enclosed by the graphs of $y = \sqrt{4x - x^2}$ and $y = \frac{x}{2}$?

(A) 1.707 (B) 2.829 (C) 5.389 (D) 8.886 (E) 21.447

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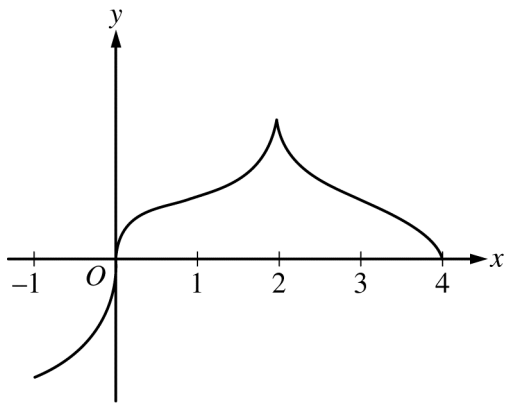

Graph of f'

82. The graph of f', the derivative of f, is shown above. The line tangent to the graph of f' at $x = 0$ is vertical, and f' is not differentiable at $x = 2$. Which of the following statements is true?

(A) f' does not exist at $x = 2$.

(B) f is decreasing on the interval $(2, 4)$.

(C) The graph of f has a point of inflection at $x = 2$.

(D) The graph of f has a point of inflection at $x = 0$.

(E) f has a local maximum at $x = 0$.

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83. A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t)$ and $\frac{dx}{dt} = -10t^4 + 9t^2 + 8t$. The acceleration of the particle is zero when $t =$

(A) 0.387 (B) 0.831 (C) 1.243 (D) 1.647 (E) 8.094

---

84. The function f is continuous on the closed interval $[1, 7]$. If $\int_1^7 f(x)\,dx = 42$ and $\int_7^3 f(x)\,dx = -32$, then $\int_1^3 2f(x)\,dx =$

(A) -148 (B) 10 (C) 12 (D) 20 (E) 148

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GO ON TO THE NEXT PAGE.

85. Let $y = f(x)$ define a twice-differentiable function and let $y = t(x)$ be the line tangent to the graph of f at $x = 2$. If $t(x) \geq f(x)$ for all real x, which of the following must be true?

(A) $f(2) \geq 0$

(B) $f'(2) \geq 0$

(C) $f'(2) \leq 0$

(D) $f''(2) \geq 0$

(E) $f''(2) \leq 0$

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86. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

(A) $\lim_{x \to 2} f(x) = 0$

(B) $\lim_{x \to 2} f(x) = -\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = \infty$

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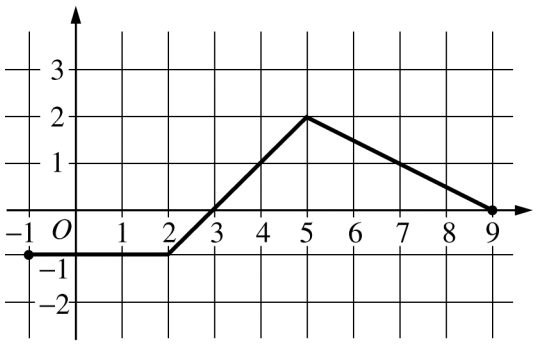
GO ON TO THE NEXT PAGE.



Graph of f

87. The graph of the piecewise linear function f is shown above. Let h be the function given by $h(x) = \int_{-1}^{x} f(t)\,dt$. On which of the following intervals is h increasing?

(A) $[-1, 3]$

(B) $[0, 5]$

(C) $[2, 5]$ only

(D) $[2, 9]$

(E) $[3, 9]$ only

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88. The first derivative of the function f is given by $f'(x) = \sin\left(x^2\right)$. At which of the following values of x does f have a local minimum?

(A) 2.507 (B) 2.171 (C) 1.772 (D) 1.253 (E) 0

---

89. If $\lim_{x \to a} f(x) = f(a)$, then which of the following statements about f must be true?

(A) f is continuous at $x = a$.

(B) f is differentiable at $x = a$.

(C) For all values of x, $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f.

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GO ON TO THE NEXT PAGE.

90. The temperature F, in degrees Fahrenheit $(°\text{F})$, of a cup of coffee t minutes after it is poured is given by $F(t) = 72 + 118e^{-0.093t}$. To the nearest degree, what is the average temperature of the coffee between $t = 0$ and $t = 10$ minutes?

(A) 93°F

(B) 119°F

(C) 146°F

(D) 149°F

(E) 154°F

---

91. If $f'(x) = \cos\left(x^2\right)$ and $f(3) = 7$, then $f(2) =$

(A) 0.241 (B) 5.831 (C) 6.416 (D) 6.759 (E) 7.241

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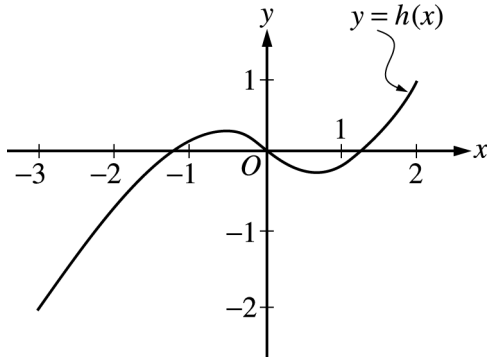

92. The graph of the function h is shown in the figure above. Of the following, which has the greatest value?

(A) Average value of h over $[-3,2]$

(B) Average rate of change of h over $[-3,2]$

(C) $\int_{-3}^{2} h(x)\,dx$

(D) $\int_{-3}^{0} h(x)\,dx$

(E) $h'(0)$

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END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

Section II: Free-Response Questions

This is the free-response section of the 2015 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION II: Free Response

2015

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

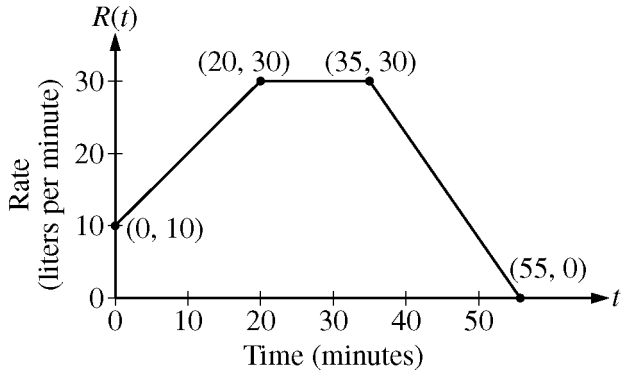
- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4KBP6-S

66

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.



1. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.

   (a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

   (b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

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Continue problem 1 on page 5.

(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

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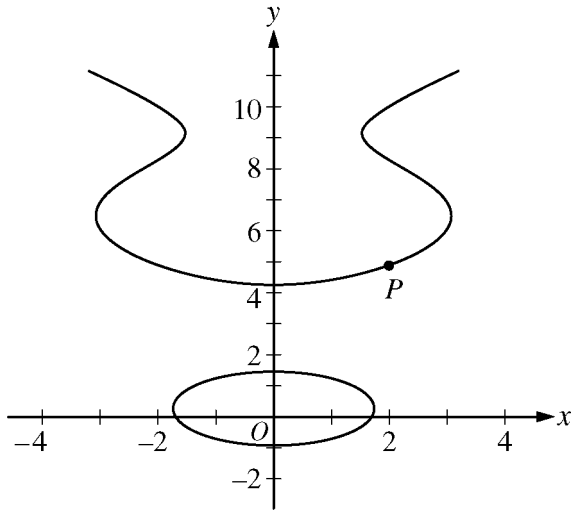

2. The graph of the equation $x^2 = -2 + y + 5\cos y$ is shown above for $y \le 11$. It is known that $\frac{dy}{dx} = \frac{2x}{1 - 5\sin y}$.

The x-coordinate of point P shown on the graph is 2.

(a) Write an equation for the line tangent to the graph at point P.

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Continue problem 2 on page 7.

(b) For $y \leq 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

(c) Find the average value of the x-coordinates of the points on the graph in the first quadrant between $y = 5$ and $y = 9$.

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

3. Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

   (a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

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Continue problem 3 on page 15.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \frac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

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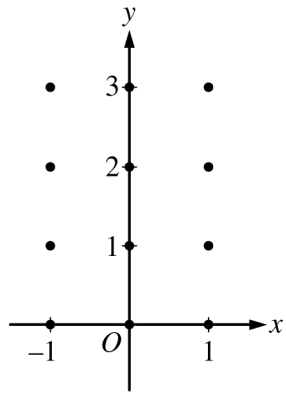
NO CALCULATOR ALLOWED

4. Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

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Continue problem 4 on page 17.

NO CALCULATOR ALLOWED

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

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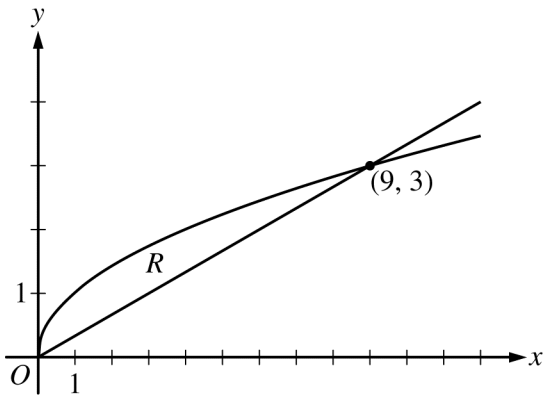

5. Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.

   (a) Find the area of region R.

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Continue problem 5 on page 19.

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

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NO CALCULATOR ALLOWED

6. Let $g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(a) Find $f(26)$.

(b) Determine the concavity of the graph of $y = f(x)$ for $x > 0$. Justify your answer.

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Continue problem 6 on page 21.

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(c) Let h be the function defined by $h(x) = x - f(x)$. Find the minimum value of h on the interval $0 \le x \le 26$.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Multiple-Choice Answer Key

The following contains the answers to
the multiple-choice questions in this exam.

Answer Key for AP Calculus AB Practice Exam, Section I

Question 1: B

Question 2: A

Question 3: E

Question 4: A

Question 5: B

Question 6: B

Question 7: D

Question 8: C

Question 9: B

Question 10: A

Question 11: C

Question 12: A

Question 13: E

Question 14: D

Question 15: C

Question 16: C

Question 17: C

Question 18: B

Question 19: C

Question 20: C

Question 21: B

Question 22: A

Question 23: B

Question 24: D

Question 25: B

Question 26: C

Question 27: B

Question 28: A

Question 76: B

Question 77: B

Question 78: D

Question 79: A

Question 80: E

Question 81: B

Question 82: C

Question 83: B

Question 84: D

Question 85: E

Question 86: A

Question 87: E

Question 88: A

Question 89: A

Question 90: D

Question 91: D

Question 92: B

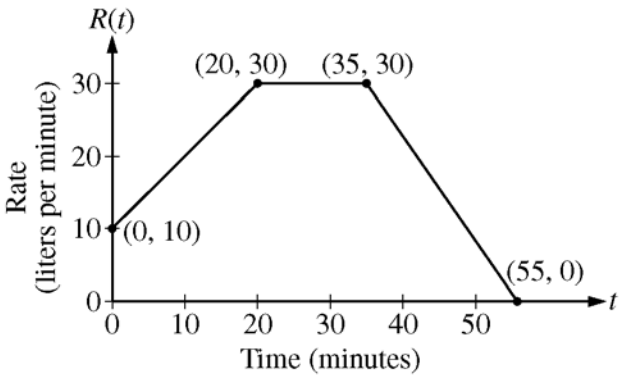
Free-Response Scoring Guidelines

The following contains the scoring guidelines
for the free-response questions in this exam.

AP® CALCULUS AB
2015 SCORING GUIDELINES

Question 1

At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.



(a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

(b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

(a) $R'(45) = \dfrac{30 - 0}{35 - 55} = -\dfrac{3}{2}$

The rate at which water is being pumped into the tank is decreasing at $\dfrac{3}{2}$ liters/min^2 at $t = 45$ minutes.

$2 : \begin{cases} 1 : R'(45) \\ 1 : \text{explanation} \end{cases}$

(b) $\displaystyle\int_0^{55} R(t)\,dt = 20 \cdot \frac{10 + 30}{2} + 15 \cdot 30 + \frac{1}{2} \cdot 20 \cdot 30$

$= 400 + 450 + 300 = 1150$

$2 : \begin{cases} 1 : \text{sum of areas} \\ 1 : \text{answer} \end{cases}$

(c) $\text{Amt} = 100 + 1150 - \displaystyle\int_{10}^{55} 10e^{(\sin t)/10}\,dt$

$= 1250 - 450.275371 = 799.725$ (or 799.724)

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression for water in the tank} \\ 1 : \text{answer} \end{cases}$

(d) $R(45) = 15$

$D(45) = 10.88815$

At time $t = 45$ minutes, the rate of water pumped into the tank is greater than the rate of water draining from the tank. Therefore, the amount of water in the tank is increasing at time $t = 45$ minutes.

$2 :$ answer with justification

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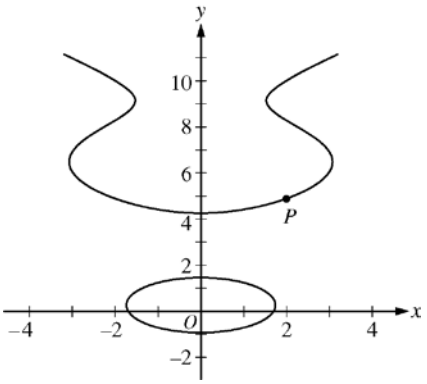
Question 2

The graph of the equation $x^2 = -2 + y + 5\cos y$ is shown above for $y \le 11$. It is known that $\frac{dy}{dx} = \frac{2x}{1 - 5\sin y}$. The x-coordinate of point P shown on the graph is 2.



(a) Write an equation for the line tangent to the graph at point P.

(b) For $y \le 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

(c) Find the average value of the x-coordinates of the points on the graph in the first quadrant between $y = 5$ and $y = 9$.

(a) $x = 2 \Rightarrow y + 5\cos y = 6 \Rightarrow y = 4.928387$

$\left.\frac{dy}{dx}\right|_{(x, y)=(2, 4.928387)} = 0.679831$

An equation for the line tangent to the graph at point P is $y = 0.680(x - 2) + 4.928$ (or $y = 0.679(x - 2) + 4.928$).

3 : { 1 : y-coordinate; 1 : slope; 1 : tangent line equation }

(b) $1 - 5\sin y = 0$

$y = 0.201, 6.485$ (or 6.484), 9.223

3 : { 1 : sets $1 - 5\sin y = 0$; 2 : y-coordinates }

(c) $x = \sqrt{y + 5\cos y - 2}$

Average value $= \frac{1}{4}\int_5^9 \sqrt{y + 5\cos y - 2}\, dy = 2.550$

3 : { 1 : integrand; 1 : limits and constant; 1 : answer }

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2015 SCORING GUIDELINES

Question 3

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

(a) $a(4) \approx \dfrac{10-5}{5-3} = \dfrac{5}{2}$ ft/sec^2

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) $\int_0^{12} k(t)\,dt \approx (5)(3) + (10)(2) + (20)(3) + (24)(4) = 191$ feet

This approximation is an overestimate since a right Riemann sum is used and the function k is increasing.

$3 : \begin{cases} 1 : \text{right Riemann sum} \\ 1 : \text{approximation with units} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) $s(12) = 5 + \int_0^{12} n(t)\,dt$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $n'(t) = (150)(-1)(t+3)^{-2} - 50e^{-t}(-1)$

$= -\dfrac{150}{(t+3)^2} + 50e^{-t}$

$2 : n'(t)$

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Question 4

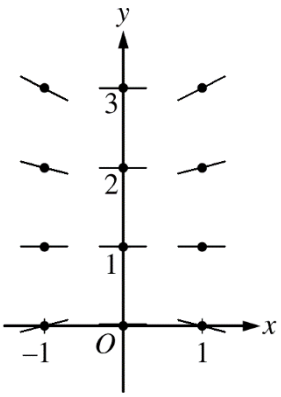
Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{other slopes} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(1,\,3)} = \frac{(1)(2)}{4} = \frac{1}{2}$

An equation for the line tangent to the graph of f at the point $(1, 3)$ is $y = \frac{1}{2}(x-1) + 3$.

$f(1.4) \approx \frac{1}{2}(0.4) + 3 = 3.2$

$2 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = \frac{x(y-1)}{4}$

$\int \frac{dy}{y-1} = \int \frac{x}{4}\,dx$

$\ln|y-1| = \frac{x^2}{8} + C$

$\ln 2 = \frac{1}{8} + C \Rightarrow C = \ln 2 - \frac{1}{8}$

Because $f(1) = 3,\ y > 1,$ so $|y-1| = y-1$.

$\ln(y-1) = \frac{x^2}{8} + \ln 2 - \frac{1}{8}$

$y = 2e^{(x^2-1)/8} + 1$ for all x

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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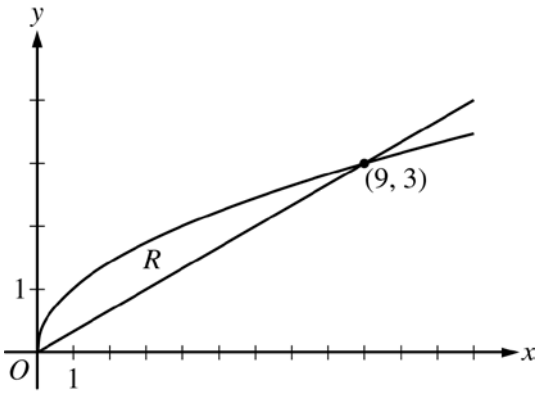
Question 5

Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.



(a) Find the area of region R.

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

(a) $\text{Area} = \displaystyle\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{6}x^2\right]_0^9$

$= \frac{2}{3} \cdot 27 - \frac{1}{6} \cdot 81 = \frac{9}{2}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) $\text{Volume} = \pi \displaystyle\int_0^9 \left[\left(4 - \frac{x}{3}\right)^2 - \left(4 - \sqrt{x}\right)^2\right] dx$

3 : { 2 : integrand; 1 : limits and constant }

(c) Consider the function $D(x) = \sqrt{x} - \frac{x}{3}$.

$D'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3} = \frac{1}{2\sqrt{x}} - \frac{1}{3}$

$D'(x) = 0 \Rightarrow x = \frac{9}{4}$

x	$D(x)$	Distance between graphs
0	0	0
$\frac{9}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
16	$-\frac{4}{3}$	$\frac{4}{3}$

The maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$ is $\frac{4}{3}$.

3 : { 1 : sets $D'(x) = 0$; 1 : identifies $x = \frac{9}{4}$ as a candidate; 1 : answer and justification }

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Question 6

Let $g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(a) Find $f(26)$.

(b) Determine the concavity of the graph of $y = f(x)$ for $x > 0$. Justify your answer.

(c) Let h be the function defined by $h(x) = x - f(x)$. Find the minimum value of h on the interval $0 \leq x \leq 26$.

(a) $f(26) = \int_0^{26} 4(x+1)^{-2/3}\,dx = \left[4 \cdot 3(x+1)^{1/3}\right]_{x=0}^{x=26}$
$= 36 - 12 = 24$

3 : { 2 : antiderivative; 1 : answer }

(b) $f'(x) = g(x) = 4(x+1)^{-2/3}$

$f''(x) = 4\left(-\frac{2}{3}\right)(x+1)^{-5/3} = -\frac{8}{3(x+1)^{5/3}}$

$f''(x) < 0$ for $x > 0$.

The graph of $y = f(x)$ is concave down for $x > 0$.

3 : { 1 : $f'(x)$; 1 : $f''(x)$; 1 : answer with justification }

(c) $h'(x) = 1 - f'(x) = 1 - 4(x+1)^{-2/3}$

$h'(x) = 0 \Rightarrow 1 = \frac{4}{(x+1)^{2/3}} \Rightarrow x = 7$

$h(7) = 7 - \int_0^7 4(x+1)^{-2/3}\,dx = 7 - \left[4 \cdot 3(x+1)^{1/3}\right]_{x=0}^{x=7}$
$= 7 - [24 - 12] = -5$
$h(26) = 26 - 24 = 2$

x	$h(x)$
0	0
7	−5
26	2

The minimum value of h on the interval $0 \leq x \leq 26$ is -5.

3 : { 1 : sets $h'(x) = 0$; 1 : identifies $x = 7$ as a candidate; 1 : answer with reason }

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Scoring Worksheet

The following provides a scoring worksheet and conversion table used for calculating a composite score of the exam.

2015 AP Calculus AB Scoring Worksheet

Section I: Multiple Choice

_____	×	1.2000	=	_____
Number Correct (out of 45)				Weighted Section I Score (Do not round)

Section II: Free Response

Question 1	_____	×	1.0000	=	_____
	(out of 9)				(Do not round)
Question 2	_____	×	1.0000	=	_____
	(out of 9)				(Do not round)
Question 3	_____	×	1.0000	=	_____
	(out of 9)				(Do not round)
Question 4	_____	×	1.0000	=	_____
	(out of 9)				(Do not round)
Question 5	_____	×	1.0000	=	_____
	(out of 9)				(Do not round)
Question 6	_____	×	1.0000	=	_____
	(out of 9)				(Do not round)

Sum = _____
Weighted Section II Score (Do not round)

Composite Score

_____	+	_____	=	_____
Weighted Section I Score		Weighted Section II Score		Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus AB

Composite Score Range	AP Score
70-108	5
58-69	4
46-57	3
38-45	2
0-37	1

AP Calculus AB

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